time. The temperature is shown to remain approximately constant during the low-gravity coast. At the restart of the second-stage engine (also the restart of propellant draining), the outlet temperature increases rapidly, indicating that engine reignition results in a rapid thermal restratification.

Conclusions

Numerical modeling methods have advanced to a point where a combined propellant-sloshing and thermal-stratification analysis is technically and financially feasible with existing software and computer hardware. A mission-long solution for a simplified cryogenic upper-stage propellant tank with a highly varying acceleration environment and heating rates presents no significant problems. Results indicate a nonintuitive liquid temperature distribution and slosh due to buoyancy-induced effects in the variable-gravity environment. Even with no propellant slosh, the draining of propellant during engine burn can distort the liquid isothermal surfaces, resulting in earlier ingestion of warmer propellant than would be predicted by a simplified analysis. Although the relatively small temperature variations (<2°R) would not affect engine operation significantly, the resulting low-gravity slosh will affect vehicle control-system requirements.

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Maximum ΔV in the Aerogravity Assist Maneuver

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Nomenclature

L/D = aerodynamic lift-to-drag ratio
r = pericenter radius, km
V = velocity of vehicle, km/s
v = normalized initial velocity at infinity
w = normalized final velocity at infinity
ΔV = change in velocity magnitude, km/s
Δv = normalized change in velocity magnitude
δ = gravity angular deflection, rad
θ = arc length in atmosphere, rad
λ = Lagrange multiplier
μ = planet's gravitational parameter, km^3/s^2
ϕ = total rotation angle, rad

Subscripts

c = circular orbit
max = maximum value
1 = initial or approach
2 = final or exit
∞ = relative to planet at infinity

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Introduction

IN the last years, a new concept in the field of aero-assisted maneuvers has been developed. This new technique, envisaged for interplanetary missions and named AGA (aero gravity assist),[1,2] utilizes atmospheric flight in order to augment the gravitational bending of heliocentric velocity occurring during a planetary encounter. The resultant ΔV is much larger than that obtained from traditional (exoatmospheric) gravity assist.

This rather paradoxical result (an atmospheric pass, with its inevitable loss of energy, being made to increase the inertial velocity) is illustrated in Fig. 1. The spacecraft enters the planet sphere of influence at the hyperbolic excess velocity following an approach asymptote with pericenter within the atmosphere. The atmospheric flight is controlled by aerodynamic lift to maintain nearly constant altitude. This circular trajectory needs very high values of lift, directed downwards, to counteract the centrifugal force. After the atmospheric turn through an angle, the spacecraft follows the leaving asymptote with a final velocity that, due to drag force, will be smaller than the incoming velocity. It is expected that this reduction will be minimized with appropriate vehicles, such as hypersonic waveriders.[3,4] The triangles of velocities shown at the bottom of the figure demonstrate that, in spite of the reduction in V_∞, the ΔV with such an AGA maneuver is greater than that obtained with gravity assist alone.

It is well known that the maximum ΔV achieved in a gravity assist maneuver is equal to the circular velocity at the hyperbola pericenter. One may ask: given the aerodynamic performance L/D of a vehicle and an approach velocity V_∞, what will be the maximum ΔV in an aerogravity assist maneuver? It is the goal of this Note to give the answer to this question.

Maximum ΔV

The dynamics of an aerogravity assist vehicle can be found in the literature[4] and will not be reviewed here. With the assumption of constant L/D, the final velocity at infinity is

$$V_{\infty 2} = \left[V_{\infty 1}^2 \exp\left\{\frac{-2\theta}{L/D}\right\} + \frac{\mu}{r}\left(\exp\left\{\frac{-2\theta}{L/D}\right\} - 1\right)\right]^{\frac{1}{2}} \tag{1}$$

and the total turning angle

$$\phi = \sin^{-1}\frac{1}{1 + r\left(V_{\infty 1}^2/\mu\right)} + \frac{1}{2}\frac{L}{D}\,\ell n\frac{1 + r\left(V_{\infty 1}^2/\mu\right)}{1 + r\left(V_{\infty 2}^2/\mu\right)} + \sin^{-1}\frac{1}{1 + r\left(V_{\infty 2}^2/\mu\right)} \tag{2}$$

is the sum of the gravity turn approach angle δ_1, the arc length θ in the atmosphere, and the gravity turn exit angle δ_2. Since μ/r is the square of the circular velocity at the pericenter, Eqs. (1) and (2) become

$$w = \sqrt{\exp\left\{-\frac{2\theta}{L/D}\right\}(1 + v^2) - 1} \tag{3}$$

$$\phi = \sin^{-1}\frac{1}{1 + v^2} + \frac{1}{2}\left(\frac{L}{D}\right)\ell n\frac{1 + v^2}{1 + w^2} + \sin^{-1}\frac{1}{1 + w^2} \tag{4}$$

where we have introduced the normalized approach and exit hyperbolic excess velocities $v = V_{\infty 1}/V_c$, $w = V_{\infty 2}/V_c$.

The amount of change in heliocentric velocity occurring in an AGA maneuver can be obtained from the corresponding triangle in Fig. 1. After normalizing, we obtain

$$\Delta v = \sqrt{v^2 + w^2 - 2vw\cos\phi} \tag{5}$$

We are interested in achieving the maximum Δv for a given v. In mathematical terms, the problem is formulated as: Find the extreme values of

$$F(w, \phi) = w^2 - 2vw\cos\phi \tag{6}$$

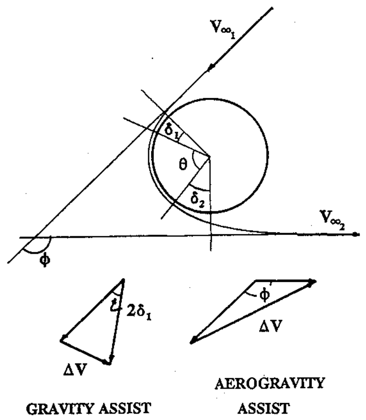

Fig. 1 Total bending angle in aerogravity maneuver. Triangles of velocities: exoatmospheric (left) and atmospheric (right) trajectories.

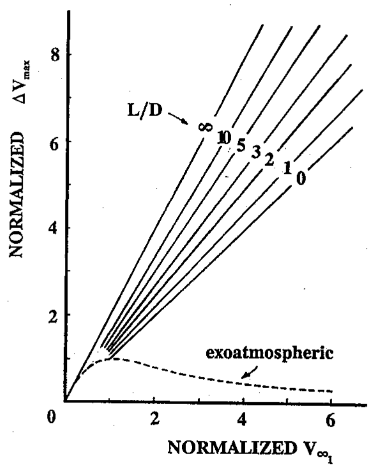

Fig. 2 Variation in maximum ΔV with initial velocity at infinity and aerodynamic performance L/D.

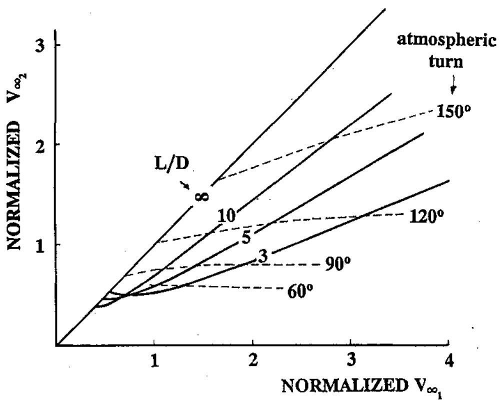

Fig. 3 Final velocity and atmospheric arc required for maximum ΔV.

with the side condition (4):

$$f = -\phi + \sin^{-1}\frac{1}{1+v^2} + \frac{1}{2}\frac{L}{D}\ln\frac{1+v^2}{1+w^2} + \sin^{-1}\frac{1}{1+w^2} = 0 \tag{7}$$

The solution is obtained via the Lagrange multiplier method:

$$\frac{\partial F}{\partial w} + \lambda\frac{\partial f}{\partial w} = 0 \tag{8}$$

$$\frac{\partial F}{\partial \phi} + \lambda\frac{\partial f}{\partial \phi} = 0 \tag{9}$$

With appropriate handling, and removing λ, we obtain

$$w - v\cos\phi - \frac{vw^2\sin\phi}{1+w^2}\left(\frac{L}{D} + \frac{2}{\sqrt{(1+w^2)^2-1}}\right) = 0 \tag{10}$$

This equation shows the relation between w and ϕ that optimizes Δv. Solving the problem requires, then, the solution of Eqs. (7) and (10). Computed results are shown in Figs. 2 and 3. Figure 2 shows normalized ΔV_{max} as a function of normalized $V_{\infty 1}$ for the whole range of L/D, and Fig. 3 illustrates the corresponding values of normalized $V_{\infty 2}$ and atmospheric turn θ.

Discussion

In the range of interest (i.e., $v > 1$) the maximum velocity change is equal to $V_{\infty 1}$ for $L/D = 0$ and equal to $2V_{\infty 1}$ for $L/D = \infty$. The interpretation of these facts is clear: the value $L/D = 0$ means a very strong (infinite) drag force; the atmospheric flight is then equivalent to an instantaneous negative impulse that reduces $V_{\infty 2}$ to zero. (A greater reduction would imply the aerocapture of the vehicle, and then we could not speak of AGA.) Therefore, the change in velocity is $V_{\infty 1}$, and the maneuver is carried out at $\theta = 0$. Note that the atmospheric turn θ must have an upper bound

$$\theta < \frac{1}{2}\frac{L}{D}\ln(v^2+1) \tag{11}$$

as can be deduced from Eq. (3), in order to get $w > 0$.

On the other hand, $L/D = \infty$ means an atmospheric flight without drag reduction; we can then turn $V_{\infty 1}$ through an angle $\phi = 180$ deg and get the maximum change $2V_{\infty 1}$. For any other value of L/D the ratio $\Delta V_{max}/V_{\infty 1}$ is nearly constant. We may use the empirical rule (for $2 < L/D < 23$)

$$\Delta V_{max} = V_{\infty 1}\left(1 + \frac{\ln(L/D)}{\pi}\right) \tag{12}$$

which is accurate to better than 5%.

For values of normalized $V_{\infty 1}$ smaller than 1, the angle θ required for ΔV_{max} may be negative; in such circumstances the atmospheric flight is absent, and the maximum change corresponds to a gravity-assist-only (exoatmospheric) maneuver. In this case, there is no drag reduction and $V_{\infty 1} = V_{\infty 2}$ (see Fig. 3).

Conclusions

The ΔV_{max} achieved in an AGA maneuver depends upon the aerodynamic parameter L/D, and its value is limited within the range from $V_{\infty 1}$ to $2V_{\infty 1}$. A simple, approximate formula is given.

The potential benefits of ΔV_{max} maneuvers remain in transferring this change of velocity in such a way that maximizes some parameter of the postencounter orbit: inclination, aphelion radius, etc. For example, a ΔV_{max} AGA maneuver in Earth's atmosphere can send a vehicle ($L/D = 3$) to Jupiter's orbit when the entry heliocentric velocity in our planet's sphere of influence is as low as 17 km/s.

It is important to notice that it is not mandatory to use vehicles with extreme L/D ratios, such as waveriders, in order to take advantage of the AGA concept. A vehicle with low lift capacity can augment the gravitational bending of $V_{\infty 1}$ through a maneuver similar to (but less severe than) aerocapture.

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